

Experiment Design:

In Experimental design, we use "Plan" to collect data relevant to problem to make valid & effective inference about the stated problem.

By "Plan" means:

- Allocation of treatment, whose effects are to be studied in the relevant problem.
- Allocation of experimental layout
- And Collection of data

⇒ Objectives:

- 1- To get maximum information in minimum expenditure & time.
- 2- By using Exp. techniques, we can avoid Systematic errors
- 3- We can explain outcomes logically & critically

⇒ Basic Principles:

- 1- Randomization
- 2- Replication
- 3- Local Control

RANDOMIZATION

Randomization is the first principle.

In Randomization, "Every treatment has a same chance to be selected in Experimental Unit."

⇒ Treatment:

"It is a condition, whose effect to be studied in the problem."

⇒ Exp. unit:

"It is a smallest (unit) division of Experimental (unit) layout/material."

By using Randomization, we remove the Biasness from the data.

Randomization is done by phying cards, lottery system, random number table and lucky draw.

REPLICATION

Replication is the second principle.

In Replication, "we repeat the basic experiment more than one time"

⇒ Basic experiment:

"It is a complete run of all the treatments."

As we know, All the experiments, individuals or plots are not similar in characteristics.

To avoid this error, by using Replication in a single use of basic experiment is called Replicate."

LOCAL CONTROL

It is observed that Randomization & Replication sometime failed to control the extraneous variation when there is need to refine an Experimental technique by introducing local control. Local Control means "Balancing, Blocking and Grouping."

⇒ Balancing:

The agreement between treatment & Exp. unit" (or)

"The agreement of allocation of the treatment between Experimental units."

⇒ Blocking:

"In blocking, the allocation of treatment in such a way that every replication should be in homogenous groups."

By using local control, we remove the variation from exp. unit, to get more and effective outcome.

K = Treatment = 4

R ₁	{	A	B	C	D	} Replications = 4 Experimental unit
R ₂	{	C	A	D	B	
R ₃	{	B	B	C	A	
R ₄	{	A				

• In CR-Design, Exp units combine to form Replications.

• But In RCBD-Designs, R = B
Replication refers to Block

Complete Randomized Design or (CR)

- It is the simplest type of Basic design.
- It is based on Randomization.
- "In CR-Design, Randomization is Un-restricted which means, any number of treatment at any number of times can be used in any number of exp. units."
- It is widely used in laboratory experiments, where we have homogenous material and some of the treatments are destroyable or fail to respond.

For weight gain	Dog Breed
diet - A	1
diet - B	2
diet - C	3

Replication	Diet	Dog Breed
(I) R-I	A	1
	B	2
	C	3

R-II	{	A	→	1	unconditional			
		B	→	2	R-I	A	B	C
		C	→	3	R-II	A	A	C
					R-III	A	B	B

R-III	Diet	Dog Breed

STAT-402

Question no 1:

(Related to CR-Design)

An experiment was conducted to compare the yield of 3 varieties of Potatoes. Each variety was assigned at random to equal size plot four times. The yield is as follow

Varieties:

$A, B, C = 3 \rightarrow$ Treatments

	T_1 A	T_2 B	T_3 C	
t_1	23	18	16	
t_2	26	28	25	
t_3	20	17	12	
t_4	17	21	14	Grand Total
Total =	86	84	67	= 237

Test the hypothesis of 3 varieties of potatoes are not different in the yielding capabilities?

Solution:-

1- Hypothesis:

$$H_0 = \mu_A = \mu_B = \mu_C$$

$H_1 =$ At least one is different
or

$$H_1 \neq \mu_A \neq \mu_B \neq \mu_C$$

2- Level of Significance:

$$\alpha = 0.05$$

$$\text{or } 0.01$$

$$\text{or } 0.025$$

3. Test Statistics,

$$F = \frac{S_t^2}{S_e^2}$$

where S_t^2 = mean square of treatment

S_e^2 = mean square error

s^2 = variance

4. Computation:

$$CF = \frac{(G)^2}{n}$$

where:

G = grand total

n = no. of values

$$CF = \frac{(G)^2}{n}$$

$$\text{Correct Factor} = \frac{(237)^2}{12}$$

$$= \frac{56169}{12}$$

$$CF = 4680.15$$

$$TSS = \sum \sum Y_{ij}^2 - CF$$

$$= 23^2 + (26)^2 + (20)^2 + \dots (14)^2 - 4680$$

$$= 4953 - 4680$$

$$TSS = 272.25$$

$$\text{Treat SS} = \frac{\sum T_i^2}{r} - CF$$

$\therefore r = \text{Replications}$

$$= \frac{(86)^2 + (84)^2 + (87)^2}{4} - 4680$$

$$\text{Treat SS} = 54.50$$

$$ESS = TSS - \text{Treat SS}$$

$$= 272.50 - 54.50$$

$$ESS = 217.75$$

ANOVA TABLE (Analysis of Variance)

Source of Variance	Degree of Freedom	Sum of Square	Mean Square	Formula
SSV	df	SS	MS	F
Treatment	$K-1 = ?$ $3-1 = 2$	54.50	$\frac{SS}{df} = \frac{54.50}{2} = 27.25$	$\frac{27.25}{24.19} = 1.13$
Error	$11-2 = 9$	217.75	$\frac{SS}{df} = \frac{217}{9} = 24.19$	
Total	$n-1 = ?$ $12-1 = 11$	272.25		

5. Critical Region:

$$F \geq F_{\alpha, (v_1, v_2)}$$

$$1.13 \geq F_{0.05} (2, 9)$$

F-distribution = level of significance

$$v_1 = \text{t df} = 2$$

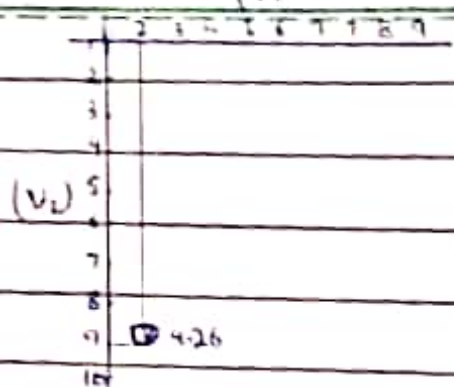
$$v_2 = \text{e df} = 9$$

$$1.13 \geq 4.26$$

$$\alpha = 0.05$$

$$\alpha = 0.01$$

$$\alpha = 0.025$$



6. Conclusion:

When the condition is false,
Accept H_0 (or Type I error)

When the condition is true, Reject H_0

As all the calculated values falls in acceptance region, So we accept H_0 .

And conclude that All the varieties of potatoes have same yielding capacity at 5% level of Significance.

Experimental layout of CR-Design:

Let we have four treatments, A, B, C, D which are used in four Replications:

A	B	C	D
C	B	D	A
A	C	D	B
B	B	C	A

* Side Information:-

⇒ Testing a hypothesis:

"A procedure which enable us whether to except or reject any statement or assumption about the population parameter, which may or may not be True."

Hypothesis may be: e.g → All diets have same effect

- Null hypothesis (H_0) → depends on Basic hypothesis
- Alternative hypothesis (H_1) → (depends) opposite to null hypothesis
e.g All diets don't have same effect

μ = average

$$H_0 = \mu_A = \mu_B = \mu_C$$

$$H_1 = \mu_A \neq \mu_B \neq \mu_C$$

⇒ Level of Significance:

"The probability of Rejecting H_0 when it is in fact True (H_0)."

RANDOMIZED COMPLETE BLOCK DESIGN (RCBD)

- In Randomized Complete Block Design (RCBD) we control one way variation by introducing Block in RCBD.
- In RCBD, we divide the experimental material into relatively homogenous g Blocks.
- Homogenous Groups/Blocks means the assign material are relatively same.
- Every block has a complete set of treatments, which is assigned by Randomization. And this randomization is restricted under the condition "Every Treatment appears single time in each replication. Every time a new randomization is made for every Block."

Experimental layout of RCBD:

+ Numerical:

4 Varieties of wheat were tried in Randomized Complete Block Design in 4 Replications. Yield in kg is shown:

Replications	Varieties				Total
	V ₁	V ₂	V ₃	V ₄	
R-I	2	5	4	1	12
R-II	2	3	3	1	9
R-III	4	6	6	2	18
R-IV	1	4	2	3	10
$\Sigma T =$	9	18	15	7	49

\Rightarrow Grand Total (G)

Test the hypothesis that both varieties have same means.

Solution:-

1. Hypothesis:

$$H_0 = \mu_A = \mu_B = \mu_C$$

$$H_1 \neq \mu_A \neq \mu_B \neq \mu_C$$

H_1 atleast one is different

2. Level of Significance:

$$\alpha = 0.01$$

3. Test Statistic:

$$F = \frac{s^2_t}{s^2_e}$$

4. Computation:

$$C.F = \frac{G^2}{Nk}$$

$$= \frac{(49)^2}{4 \times 4} = 150.06$$

$$TSS = \sum \sum Y_{ij}^2 - C.F$$

$$= (2)^2 + (5)^2 + (4)^2 + (1)^2 + (2)^2 + (3)^2 + \dots + (3)^2 - 150.06$$

$$= 191 - 150.06$$

$$= 40.9$$

$$\text{Treat SS} = \frac{\sum T_i^2}{n} - C.F$$

$$= \frac{(20)^2 + (18)^2 + (15)^2 + (7)^2}{4} - 150.06$$

$$= 19.69$$

$$\text{Block SS} = \frac{\sum B_i^2}{k} - C.F$$

$$= \frac{(12)^2 + (9)^2 + (8)^2 + (10)^2}{4} - 150.06$$

$$= 12.19$$

$$\begin{aligned}\text{Error SS} &= \text{TSS} - \text{TreatSS} - \text{Block SS} \\ &= 40.9 - 19.69 - 12.19 \\ \text{ESS} &= 9.06\end{aligned}$$

ANOVA TABLE

Source	df (k-1)	SS	MS SS/df	Formula $F = \frac{S^2_t}{S^2_e}$
Treatment	4-1=3	19.69 40.9	19.6/3 = 6.56	$F = \frac{6.56}{1.006}$
Block	4-1=3	12.19	12.19/3 = 4.06	
Error	3-15+3=9	9.06	9.06/9 = 1.006	$F = 6.52$
Total	16-1=15 n-1 ↗	40.94		

$$\text{Computation} = 6.52$$

5. Critical Region:

$$F \geq F_{\alpha} (V_1, V_2)$$

$$6.52 \geq F_{0.01} (3, 9)$$

$$6.52 \geq 6.99$$

6. Conclusion:

*(We accept H_0 as condition is False.) As all the calculated values falls in acceptance region, so we conclude that all varieties of Wheat have same average. there is no difference in yielding capacity at level of significance of 0.01 or 1%.

1- Multiple Comparison Test

If F test, after the Anova table, rejects the null hypothesis (H_0) and then we conclude that the mean of K-treatments are not equal. It would not sufficient for the experiment (then) in this situation, we would like to find the significant (figures) ^{pairs} of K-treatment. For this comparison, we use Multiple Comparison Test.

2- Least Significant difference Test (LSDT)

When the H_0 is rejected, after the Anova Table, there is need to find the significant pairs by comparing two treatments means \bar{X}_i and \bar{X}_j , where $\bar{X}_i \neq \bar{X}_j$.

For each pair, we need $\frac{1}{2}k(k-1)$ by using (simple) two samples T-test. This procedure needs many decisions to overcome this difficulty, we can calculate a least difference of the two treatment means, with comparing a smallest difference is given by

$$LSD = t_{\alpha/2}(edf) \sqrt{\frac{2MSE}{r}} \quad \therefore \text{for equal no. of Replications}$$

is called Least Significant difference Test

Where $MSE = MSE$ is the Mean Square Error & " n " is the number of Replications. In equal sizes and $\frac{t_{\alpha}}{2}$ is the value for the

t at α percent level of significance

If the absolute difference of two-treatment means exceeds the LSD value, then this pair of treatment is considered as significant.

Numerical :

$$LSD = \frac{t_{\alpha} (edf)}{2} \sqrt{\frac{2MSE}{n}}$$

$$LSD = \frac{t_{0.05} (9)}{2} \sqrt{\frac{2(\frac{1.006}{4})}{4}}$$

$$\therefore \frac{t_{0.05}}{2} = t_{0.025} \times 9 = 2.26 \rightarrow \text{from t-distribution Table}$$

$$LSD = 2.26 \left(\frac{0.709}{4} \right)$$

$$LSD = 1.602$$

(Average) *

Arranging treatments means in Ascending order:

\bar{X}_4 \bar{X}_{31} \bar{X}_3 \bar{X}_2 1.752.25

3.75

~~4.5~~ 4.5

$$\bar{X}_1 - \bar{X}_4 = 2.25 - 1.75 = 0.5 \quad \therefore 0.5 < 1.602 \text{ (insignificant)}$$

$$\bar{X}_3 - \bar{X}_4 = 4.5 - 1.75 = 2 \quad \therefore 2 > 1.602 \text{ (significant)}$$

$$\bar{X}_3 - \bar{X}_1 = 3.75 - 2.25 = 1.5 \quad 1.5 < 1.602 \text{ (insignificant)}$$

$$\bar{X}_1 - \bar{X}_2 = 2.25 - 4.5 = -2.25 \quad \text{significant}$$

$$\bar{X}_2 - \bar{X}_3 = 4.5 - 3.75 = 0.75$$

$$X_1 - X_4 < 1.60$$

$$X_3 - X_4 > 1.60$$

$$X_2 - X_{34}$$

$$(\bar{X}_4, \bar{X}_3) (\bar{X}_4, \bar{X}_2) (\bar{X}_1, \bar{X}_2)$$

These are the Significant pairs.

For Unequal Replications:

$$LSD = t_{\alpha/2} (v_2) \sqrt{2s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} + \dots + \frac{1}{n_k} \right)}$$

Block or Replication SS.

Treatment SS.

LATIN SQUARE DESIGN

In Latin Square Design, we control the two-sided variations by introducing Blocks. This Blocking is mutually perpendicular means from row-side & column-side. There is a balanced agreement b/w experimental material & exp. unit. Each Row & Column have the homogeneous material within the groups.

Numerical :

5 fertilizers A, B, C, D, E were tested by arranging Plants in LS-Design in a field. The rows & columns in the table are rows & columns in the field. The yield in bushel is given below:

Rows	Columns					Total	
	1	2	3	4	5		
R-1	B 4.9	D 6.4	E 3.3	A 9.5	C 11.8	35.9	
R-2	C 9.3	A 4.0	B 6.2	E 5.1	D 5.4	30	
R-3	D 7.6	C 15.4	A 6.5	B 6.0	E 4.6	40.1	
R-4	E 5.3	B 7.6	C 13.2	D 8.6	A 4.9	39.6	
R-5	A 9.3	E 6.3	D 11.8	C 15.9	B 7.6	50.9	
Total	C 36.4	G 37.7	C ₃ 41	C ₄ 45.1	C ₅ 34.3	196.5	Grand Total

Analysis the data for evidence at 5% level of significance, that the mean yields are not equal for the 5 fertilizers. Also pick up the best fertilizer.

Solution:

1. Hypothesis:

$$H_0 = \mu_A = \mu_B = \mu_C = \mu_D = \mu_E$$

H_1 at least 1 is different

2. level of Significance: 3. Test Statistics:

$$\alpha = 0.05$$

or

5%

$$F = \frac{S^2_t}{S^2_e}$$

3. 4. Computation:

$$C.F = \frac{G^2}{k^2}$$

$$= \frac{(196)^2}{(5)^2}$$

$$\frac{38612.25}{25}$$

$$= 1544.49$$

$$TSS = \sum \sum y_{ij}^2 - C.F$$

$$= (4.9)^2 + (6.4)^2 + \dots + (7.6)^2 - 1544.49$$

$$= 1829.83 - 1544.49$$

$$= 285.34$$

$$Row\ SS = \sum \frac{R_i^2}{k} - C.F$$

$$= \frac{(35.9)^2 + (30)^2 + (40.1)^2 + (39.6)^2 + (50.9)^2}{5} - 1544.49$$

$$= \frac{(1288.8 + 900 + 1608.0 + 1568.1 + 2590.8)}{5} - 1544.49$$

$$= 1591.14 - 1544.49$$

$$= 46.74$$

$$Column\ RSS = \sum \frac{G_j^2}{k} - C.F$$

$$= \frac{(36.4)^2 + (39.1)^2 + (41)^2 + (45)^2 + (34)^2}{5} - 1544.49$$

$$= \frac{1324.9 + 1576.0 + 1681 + 2034.0 + 1176.4}{5} - 1544.49$$

$$= 1558.28 - 1544.49$$

$$= 13.79$$

$$Treat\ SS = \sum \frac{T_i^2}{k} - C.F$$

Treat SS:-

A	B	C	D	E
9.5	4.9	11.8	6.4	3.3
4.0	6.2	9.3	5.4	5.1
6.5	6.0	15.4	7.6	4.6
4.9	7.6	13.2	8.6	5.3
9.3	7.6	15.2	11.8	6.3
T_1 34.2	T_2 32.3	T_3 65.6	T_4 39.8	T_5 24.6

$$\text{Treat SS} = \sum_k \frac{T_i^2}{k} - CF$$

$$= \frac{(34.2)^2 + (32.3)^2 + (65.6)^2 + (39.8)^2 + (24.6)^2}{5} - 1544.49$$

$$= \frac{8705.49}{5} - 1544.49$$

$$= 1741.098 - 1544.49 = 196.608$$

$$\text{Error SS} = \text{TSS} - \text{Treatment SS} - \text{Row SS} - \text{Col. SS}$$

$$= 285.34 - 196.608 - 46.67 - 19.02$$

$$= 28.04$$

Now ANOVA TABLE:-

Day: MTWTFSS

Sov	d.f	SS	MSS	Formula
	$(k-1=?)$		$SS/t = ?$	$1. MSS / E. MSS = ?$
Treatment	$5-1=4$	196.60	49.15	$F = 47.15 / 2.34$
Rows	4	46.67	11.66	
Columns	4	14.02	3.505	$F = 21.00$
Error	$(5-1)(5-2)$ $= 12$	28.04	2.34	
Total	$k^2-1=5^2-1$ $= 24$	285.34		

Critical Region:

$$F \geq F_{\alpha} (v_1, v_2)$$

 $v_1 = \text{column \#}$ $v_2 = \text{Row \#}$

$$F \geq F_{0.05} (4, 12)$$

$$21 \geq 3.26$$

Conclusion:

Reject H_0

AFTER MIDS**MULTIPLE COMPARISON TEST**

- 1- Least Significant difference test
- 2- Duncan's Multiple range test
- 3- Factorial Test
- 4- Orthogonal Latin square test or Graeco Latin Square Design
- 5- Randomize Block Complete Block Design

DUNCAN'S MULTIPLE RANGE TEST

- Duncan's Multiple range test is used to compare the treatment means when H_0 is rejected.
- This test is based on different significant ranges which is obtained from studentized statistics $q(p, v)$.
- where: " p " is the number of comparisons b/w the means
- and " v " is the errors degrees of freedom from ANOVA table
- then the value of $q(p, v)$ is obtained from Duncan's table
- The α percent least significant ranges are called R_p .
- where

$$R_p = q_{\alpha}(p, v) \sqrt{\frac{S_e^2}{n}}$$

with $P = 2, 3, 4, \dots, k-1, k$

To perform this test,

- 1- arrange the treatment means in increasing order of magnitude \bar{y}
- 2- Calculate the value of R_p by multiplying the Duncan's table value with mean square of error.
- 3- Compare the observed difference of largest (difference)¹ mean and smallest mean with least significant range
- 4- (R_k) and compare the observed difference of largest mean and second smallest mean with least significant Range (R_{k-1}) and so on
- 5- Continue this procedure until $k(k-1)/2$ comparisons are made.

⇒ A Treatment mean is significantly different if the observed difference is greater than the Least significant range.

Numerical:

$$Se^2 = 1.57$$

$$h = 4$$

$$RK = 4$$

$$\bar{X}_1 = 1.75$$

$$\bar{X}_2 = 3.50$$

$$\bar{X}_3 = 4.00$$

$$\bar{X}_4 = 4.75$$

$$\alpha = 0.05$$

$$edf = 15$$

Solution:

$$R_p = q_{\alpha}(P.V) \sqrt{\frac{Se^2}{Y}}$$

$$\sqrt{\frac{Se^2}{Y}} = \sqrt{\frac{1.57}{4}} = 0.6265$$

P	$q_{\alpha}(P.V)$ $q_{\alpha,0.05}(P, 15)$	$R_p = q_{\alpha}(P.V) \sqrt{\frac{Se^2}{Y}}$	
2	3.01	$(3.01)(0.6265) = 1.89$	R_2
3	3.16	$3.16 \times 0.6265 = 1.98$	R_3
4	3.25	$3.25 \times 0.6265 = 2.04$	R_4

⇒ Arrange in increasing order:-

\bar{X}_1	\bar{X}_2	\bar{X}_3	\bar{X}_4	$\therefore \frac{k(k-1)}{2}$
↓	↓	↓	↓	
1.75	3.50	4.0	4.75	$= \frac{4(4-1)}{2}$
				$= \frac{4(3)}{2}$
				$= 6$

⇒ Comparison:-

$$\left\{ \begin{array}{l} \bar{X}_4 - \bar{X}_1 = 4.75 - 1.75 = 3 > 2.04 \quad R_4 \\ \bar{X}_4 - \bar{X}_2 = 4.75 - 3.50 = 1.25 < 1.98 \quad R_3 \\ \bar{X}_4 - \bar{X}_3 = 4.75 - 4.00 = 0.75 < 1.89 \quad R_2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \bar{X}_3 - \bar{X}_1 = 4.00 - 1.75 = 2.25 > 1.89 \quad R_3 \\ \bar{X}_3 - \bar{X}_2 = 4.00 - 3.50 = 0.5 < 1.89 \quad R_2 \end{array} \right.$$

$$\left\{ \bar{X}_2 - \bar{X}_1 = 3.50 - 1.75 = 1.75 < 1.89 \quad R_2 \right.$$

Numerical:

2^2 factorial experiment with two varieties ^{manures} measures - factors carried out in a randomized Complete Block Design with 3 replications. The yield given in table:

Treatment Contributions					
Replications	V_1m_1	V_2m_1	V_3m_1	V_2m_2	Blocks
R-1	5	7	8	10	30 B_1
R-2	4	4	7	5	20 B_2
R-3	6	2	9	12	31 B_3
Total:	$T_1 = 15$	$T_2 = 15$	$T_3 = 24$	$T_4 = 27$	81 $= G$

Perform analysis of variance & test the significance of varieties & manures

Solution:

1. Hypothesis:

H_0 : there is no difference b/w treatment & combination

H_1 : there is a difference

2. level of Significance:

$$\alpha = 0.05$$

3. Test Statistics:

$$F = \frac{S^2_t}{S^2_e}$$

4. Computation:

$$C.F = \frac{(G)^2}{kn}$$

$$= \frac{(81)^2}{142}$$

$$= 546.75$$

$$TSS = \sum \sum y_{ij}^2 - C.F$$

$$= 74.25$$

$$ESS = \text{Treatment SS} = \sum \frac{T_i^2}{n} - C.F$$

$$= \frac{(\quad)^2 + (\quad)^2 + (\quad)^2 + (\quad)^2}{3} -$$

$$= 38.25$$

$$\text{Block SS} = \sum \frac{B_i^2}{rk} - C.F$$

$$= \frac{(\quad)^2 + (\quad)^2 + (\quad)^2}{4} - 81$$

$$= 18.50$$

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Day: M T W T F S

Date: ___/___/20___

$$\text{Error SS} = \text{TSS} - \text{Treat. SS} - \text{Block SS} \\ = 17.50$$

	m1	m2	Total
V1	15	15	30
V2	24	27	51
	39	42	81

$$\text{Total SS} = \sum \sum y_{ij}^2 - \text{C.F.} = 38.25$$

$$\text{Variety SS} = \frac{(30)^2 + (51)^2}{\text{average value} = 6} - \text{C.F. } 81 = 36.75$$

$$\text{Manure mss} = \frac{(39)^2 + (42)^2}{6} - \text{C.F. } 81 \\ = 0.75$$

Interaction SS =

$$\text{VMSS} = \text{Total SS} - \text{VSS} - \text{mss}$$

$$= 38.25 - 36.75 - 0.75$$

$$= 0.75$$

ANOVA TABLE

Error = total n - remaining values

ANOVA TABLE

Sor	df	SS	MS SS/df	F s^2/s^2_e
Treat	3			
Block	2			
Error	3		s^2_e	
V	1			
M	1		s^2_m	
VM	1			
Total	$n-1$ $12-1$ $= 11$			

Critical Region:

$$F \geq F_{\alpha}(V_1, V_2)$$

Conclusion:

Computation:

Radiation	Treatment Combination				Total
	a ₀ b ₀	a ₀ b ₁	a ₁ b ₀	a ₁ b ₁	
1	6	12	26	21	65
2	14	14	17	16	61
3	8	13	21	20	62
4	9	11	30	17	67
5	7	13	27	21	68
Total	44	63	121	95	323

$$CF = \frac{G^2}{n \times k}$$

$$\text{Treat SS} = \sum \frac{T_i^2}{n} - C.F$$

$$\text{Total SS} = \sum \sum y_{ij}^2 - CF$$

B	A		
	a ₀	a ₁	total
b ₀	44	121	165
b ₁	63	95	158
Total	107	216	323

$$\text{Total SS} = \frac{(44)^2 + (121)^2 + (63)^2 + (95)^2}{5} - 323$$

$$ASS = \frac{(107)^2 + (216)^2}{10} - 323$$

$$BSS = \frac{(165)^2 + (158)^2}{10} - 323$$

$$ABSS = \text{Total SS} - ASS - BSS$$

"ORTHOGONAL LATIN SQUARE DESIGN"

or

Also known as "Graeco Latin Square design"

- It controls 3 way variation.
- By superimposing two Latin designs.
- But one restriction / condition:
"Each Latin letter must appear single time with each greek letter."

A	B	C
B	C	A
C	A	B

+

α	β	γ
γ	α	β
β	γ	α

α		

Wrong

- It controls 3 sided variations.

Two Latin Square

I

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

II

α	β	γ	δ
δ	γ	α	β
γ	δ	β	α
β	α	δ	γ

Superimpose Square III

A α	B β	C γ	D δ	R_{10}
B δ	A γ	D α	C β	R_{20}
C γ	D δ	A β	B α	R_{30}
D β	C α	B δ	A γ	R_{40}
G_{10}	G_{20}	G_{30}	G_{40}	

3 way variations:

- Row side
- Column side
- Greek letter

→ Grand Total

Computation:

$$C.E = \frac{G^2}{k^2}$$

$$TSS = \sum \sum \sum Y_{ijk}^2 - C.F$$

$$TSS = \text{Row SS} = \sum \frac{R_i^2}{k} - CF$$
$$= \frac{R_1^2 + R_2^2 + R_3^2 + R_4^2}{4} - CF$$

$$\text{Col. SS} = \sum \frac{C_j^2}{k} - CF$$
$$= \frac{C_1^2 + C_2^2 + C_3^2 + C_4^2}{4} - CF$$

For Treatment SS :

Arrange the table according to alphabets:

Randomization

Day: MTWTFSS

Date: 1/12/20

then:

$$\text{Treat SS} = \sum \frac{T_i^2}{k} - CF$$

$$= \frac{T_1^2 + T_2^2 + T_3^2 + T_4^2}{k} - CF$$

Now arrange table according to greek letters:

$$\text{Greek SS} = \sum \frac{G_i^2}{k} - C.F$$

$$= \frac{\alpha_1^2 + \beta_1^2 + \gamma_1^2 + \delta^2}{4} - C.F$$

$$\text{Error SS} = \text{Total SS} - \text{Row SS} - \text{Col SS} - \text{Treat SS} - \text{Greek SS}$$

ANOVA TABL

Source	df	SS	MS	F
Treat	$k-1=3$			$\frac{St^2}{Se^2}$
Row	$k-1=3$			
Column	$k-1=3$			
Greek	$k-1=3$			
Error	$n-1=?$ $16-1=15$			
Total	$k^2-1=?$ $4^2-1=15$			

Error = Total mn sy baqi sb k total minus kany hain.

$$15 - 12 = 3$$